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Comparison of isothermal and non-isothermal pipeline gas flow models

Andrzej J. Osiadacz*, Maciej Chaczykowski

Institute of Heating and Ventilation, Warsaw University of Technology, Nowowiejska 20, 00-653 Warsaw, Poland

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Abstract

The transient flow of gas in pipes can be adequately described by a one-dimensional approach. Basic equations describing the transient flow of gas in pipes are derived from an equation of motion (or momentum), an equation of continuity, equation of energy and state equation.

In much of the literature, either an isothermal or an adiabatic approach is adopted. For the case of slow transients caused by fluctuations in demand, it is assumed that the gas in the pipe has sufficient time to reach thermal equilibrium with its constant-temperature surroundings. Similarly, when rapid transients were under consideration, it was assumed that the pressure changes occurred instantaneously, allowing no time for heat transfer to take place between the gas in the pipe and the surroundings.

For many dynamic gas applications, this assumption of a process having a constant temperature or is adiabatic is not valid. In this case, the temperature of the gas is a function of distance and is calculated using a mathematical model, which includes the energy equation.

In the paper, a comparison of different (isothermal and non-isothermal) models is presented. Practical examples have been used to emphasize differences between models.

Keywords: Gas flow; Mathematical modeling; Simulation; Gas networks

1. Introduction

It is a well-established fact that flow in gas pipelines is unsteady. Conditions are always changing with time, no matter how small some of the changes may be. When modeling systems, however, it is sometimes convenient to make the simplifying assumption that flow is steady. Under many conditions, this assumption produces adequate engineering results. On the other hand, there are many situations where an assumption of steady flow and its attendant ramifications produce unacceptable results. Dynamic models are just a particular class of a differential equation model in which time derivatives are present.

During transport of gas in pipelines, the gas stream loses a part of its initial energy due to frictional resistance which results in a loss of pressure. This is compensated for by compressors installed in compressor stations.

Compression of the gas has the undesired side effect of heating the gas. The gas may have to be cooled to prevent damage to the main transmission pipeline. If the cooler is installed, heat from the gas is passed to the air in a force draught heat exchanger in which one or more fans operate, depending on the number of compressors in service.

Cooling of the gas is desirable because it improves the efficiency of the overall compression process. As always, it is a matter of balancing capital and maintenance costs against operating costs.

2. Basic equations

The transient flow of gas in pipes can be adequately described by a one-dimensional approach. Basic equations describing the transient flow of gas in pipes are derived from an equation of motion (or momentum), an equation of continuity, equation of energy and state equation [7]. In practice, the form of the mathematical models varies with the assumptions made as regards the conditions of operation of the networks. The simplified models are obtained by neglecting some terms in the basic model as a result of a quantitative estimation of the particular elements of the equation for some given conditions of operation of the network [3,8].

2.1. Conservation of mass: continuity equation

Generally, the continuity equation is expressed in the form

$$-\frac{\partial(\rho w)}{\partial x}=\frac{\partial \rho}{\partial t} \tag{1}$$

* Corresponding author. Tel.: +48-22-660-53-11; fax: +48-22-825-29-92.
E-mail addresses: andrzej.osiadacz@is.pw.edu.pl (A.J. Osiadacz), maciej.chaczykowski@is.pw.edu.pl (M. Chaczykowski).

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Nomenclature

A	cross-section area of the pipe (m^2)
c_p	specific heat at constant pressure (J/kg K)
c_v	specific heat at constant volume (J/kg K)
D	pipe diameter (mm)
f	Fanning friction coefficient (–)
g	the net body force per unit mass (the acceleration of gravity) (m/s^2)
k	pipe roughness (mm)
k_L	heat transfer coefficient (W/m K)
L	pipeline length (m)
M	mass flow (kg/s)
$p(x)$	pressure at x (Pa)
q	the heat addition per unit mass per unit time (W/kg)
Q_n	flow (under standard conditions; flow rate Q_n is shown in the standard conditions of 273.15 K, 0.1 MPa) (m^3/h)
R	specific gas constant (J/kg K)
t	time (s)
T	gas temperature (K)
T_{soil}	soil temperature (K)
w	flow velocity (m/s)
x	spatial coordinate (m)
Z	compressibility factor (–)

Greek symbols

α	the angle between the horizon and the direction x
λ	thermal conductivity coefficient of gas (W/m K)
μ	viscosity of natural gas ($N s/m^2$)
ρ	density of gas (kg/m^3)

where w is the flow velocity, and ρ is the density of gas.

Substituting $M=\rho w A$, we have

$$-\frac{1}{A}\frac{\partial M}{\partial x}=\frac{\partial \rho}{\partial t} \tag{2}$$

where A is the cross-sectional area of the pipe, and M is the mass flow.

2.2. Newton's second law of motion: momentum equation

According to [2], the basic form of momentum equation can be expressed in the form

$$-\frac{\partial p}{\partial x}-\frac{2f\rho w^2}{D}-g\rho\sin\alpha=\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho w^2)}{\partial x} \tag{3}$$

where f is the Fanning friction coefficient, g the net body force per unit mass (the acceleration of gravity), and where α is the angle between the horizon and the direction x.

The constituent factors $(\partial/ot)(gw)$, $((2f\rho w^2)/D)$, $(\rho g \sin\alpha)$ and $(\partial/\partial x)(\rho w^2)$ define the gas inertia, hydraulic friction force, force of gravity and the flowing gas dynamic pressure, respectively.

2.3. State equation

An equation of state for a gas relates the variables p, ρ, and T. The type of equation which is commonly used in the natural gas industry is [6,11]

$$\frac{p}{\rho}=ZRT \tag{4}$$

where the deviation from the ideal gas law is absorbed in the compressibility factor Z, which is a function of p and T.

2.4. Conservation of energy

The basic form of energy equation, according to [10], is the following:

$$q\rho A\,dx=\frac{\partial}{\partial t}\left[(\rho A\,dx)\left(c_vT+\frac{w^2}{2}+gz\right)\right]+\frac{\partial}{\partial x}\left[(\rho wA\,dx)\left(c_vT+\frac{p}{\rho}+\frac{w^2}{2}+gz\right)\right] \tag{5}$$

where q is the heat addition per unit mass per unit time, T the gas temperature, and where c_v is the specific heat at constant volume.

Before going over to analysis of transient conditions, the simple case of steady-state conditions will be considered.

3. Steady-state non-isothermal model

The temperature of gas, as a function of length of the pipe, is calculated using the heat balance equation, assuming that heat transfer process is quasi-steady-state, expressed by the following equation:

$$c_pM\,dT=-k_L(T-T_{soil})\,dx \tag{6}$$

where c_p is the specific heat at constant pressure, J/kg K; M the mass flow, kg/s; k_L the heat transfer coefficient, W/m K; T the gas temperature, K; and where T_{soil} is the soil temperature, K.

The form of Eq. (6) results from the following transformations of Eq. (11). Since under steady-state conditions

$$\frac{\partial}{\partial t}\left[(\rho A\,dx)\left(c_vT+\frac{w^2}{2}+gz\right)\right]=0$$

we can write the energy equation describing the flow of gas throw the horizontal pipe in the form

$$q\rho A\,dx=\frac{\partial}{\partial x}\left[(\rho wA\,dx)\left(c_vT+\frac{p}{\rho}+\frac{w^2}{2}\right)\right]$$

The quantitative analysis of energy equation (Section 4.1) has shown that, under steady-state conditions, term (III) (spatial derivative of the kinetic energy) can be neglected in comparison with other convective terms. Substituting the enthalpy and neglecting term (V) (heat conduction through the gas along the pipeline), we finally obtain the energy equation in the form of the heat balance equation. Eq. (6) can be written in the form

$$\frac{\mathrm{d}T}{T - T_{\mathrm{soil}}} = -\frac{k_L}{c_p M}\mathrm{d}x$$

By integrating between $T(0)$, $(T_{x=0})$ and $T(x)$, $x \in (0, L]$, we get

$$\int_{T(0)}^{T(x)} \frac{\mathrm{d}T}{T - T_{\mathrm{soil}}} = -\frac{k_L}{c_p M}\int_0^x \mathrm{d}x$$

Finally,

$$T(x) = T_{\mathrm{soil}} + (T(0) - T_{\mathrm{soil}})\mathrm{e}^{-\beta x} \tag{7}$$

where $\beta = k_L/(c_p M)$.

For non-isothermal steady-state flow, pressure at a specific point of the pipe can be expressed by the following equation [1]:

$$p(x) = \sqrt{(P(0)^2) - KM^2} \tag{8}$$

where $p(0)$ is the pressure at $x=0$, Pa; M the mass flow, kg/s; and where K is the coefficient defined by the equation

$$K = \frac{ZR}{A^2}\left[\frac{4f}{D}\left(T_{\mathrm{soil}}x + \frac{T(0) - T_{\mathrm{soil}}}{\beta} - \frac{T(0) - T_{\mathrm{soil}}}{\beta}\mathrm{e}^{-\beta x}\right) - 2(T(0) - T(x))\right] \tag{9}$$

where x is the spatial coordinate, m; f the Fanning friction coefficient; Z the compressibility factor; R the specific gas constant, J/kg K; and where A is the cross-sectional area of the pipe, m^2.

Specific heat at constant pressure can vary strongly with temperature of the gas. In the gas transportation systems, the temperature ranges are modest and the value of specific heats may be assumed constant [4]. Such an assumption, however, cannot be applied with regard to the compressibility factor. Its value varies significantly with temperature and pressure of the gas and separate calculations are carried out for every discretization section of the pipeline.

3.1. Steady-state simulation

Differences between two models (isothermal and non-isothermal) were analyzed using a part of the existing real gas system, Yamal — West Europe. This gas transportation system (Fig. 1) consists of five compressor stations, installed on the Polish terrain. At each compressor station, there are installed between two and three centrifugal compressors, driven by gas turbines. For the purpose of our investigation, one pipe between two compressor stations was taken.

Calculations were carried out for the following parameters:

- pipe diameter D=1422 mm, pipe wall thickness 19.2 mm;
- pipeline length L=122 km;
- pressure at x=0 (discharge pressure) p_1=8.4 MPa;
- temperature at x=0 (discharge temperature) $T|_{x=0}$= 42.5°C;
- density of natural gas (under standard conditions) ρ_{n}=0.7156 kg/m^3;
- viscosity of natural gas μ=0.135×10^{-4} N s/m^2;
- flow (under standard conditions) Q_{n}=2 019 950 m^3/h;
- soil temperature T_{soil}=12°C;
- heat transfer coefficient k_L=25 W/m K;
- pipe roughness k=0.03 mm.

Fanning friction coefficient and compressibility factor were calculated for every discretization section of the pipeline using Nikuradse [1] and SGERG 88 [5] equations, respectively. Results of the investigations are shown in Figs. 2 and 3.

Maximum pressure difference between isothermal and non-isothermal flow is given in the following equation:

$$\delta_{\max} = \frac{p_{\mathrm{isotherm}}|_{x=J} - p_{\mathrm{n\text{-}isotherm}}|_{x=J}}{p_{\mathrm{isotherm}}|_{x=J}} \times 100\% = \frac{7.894 - 7.879}{7.894} \times 100 = 0.19\%$$

Maximum pressure difference between non-isothermal flow (without cooling system) and non-isothermal flow (with cooling system) is given in the following equation (see Table 1):

$$\delta_{\max} = \frac{p_{\mathrm{n\text{-}isotherm}}|_{x=J} - p_{\mathrm{n\text{-}iso\text{-}c}}|_{x=J}}{p_{\mathrm{n\text{-}isotherm}}|_{x=J}} \times 100\% = \frac{7.894 - 7.885}{7.894} \times 100 = 0.11\%$$

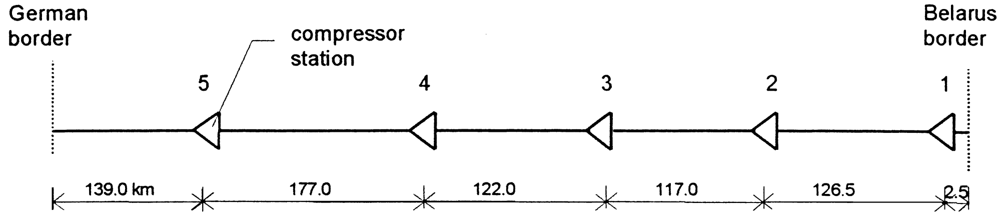

Fig. 1. Structure of gas transportation system.

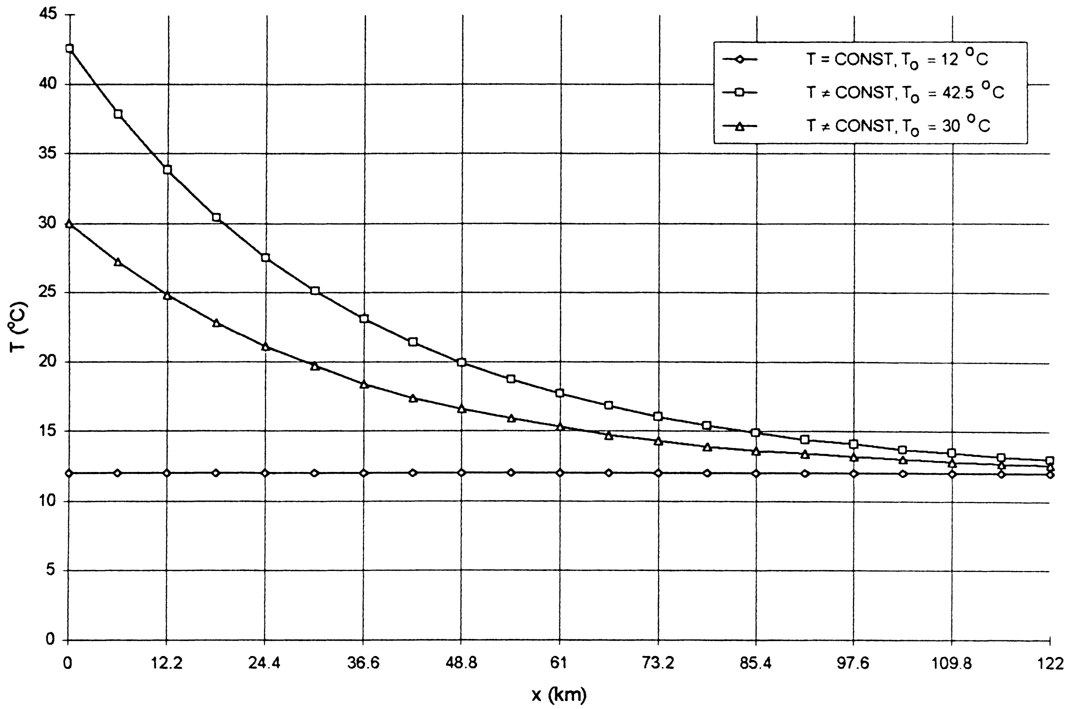

Fig. 2. Temperature profile along the pipeline for (a) T=const, T_0=12°C, (b) $T\neq$const, T_0=42.5°C, (c) $T\neq$const, T_0=30°C.

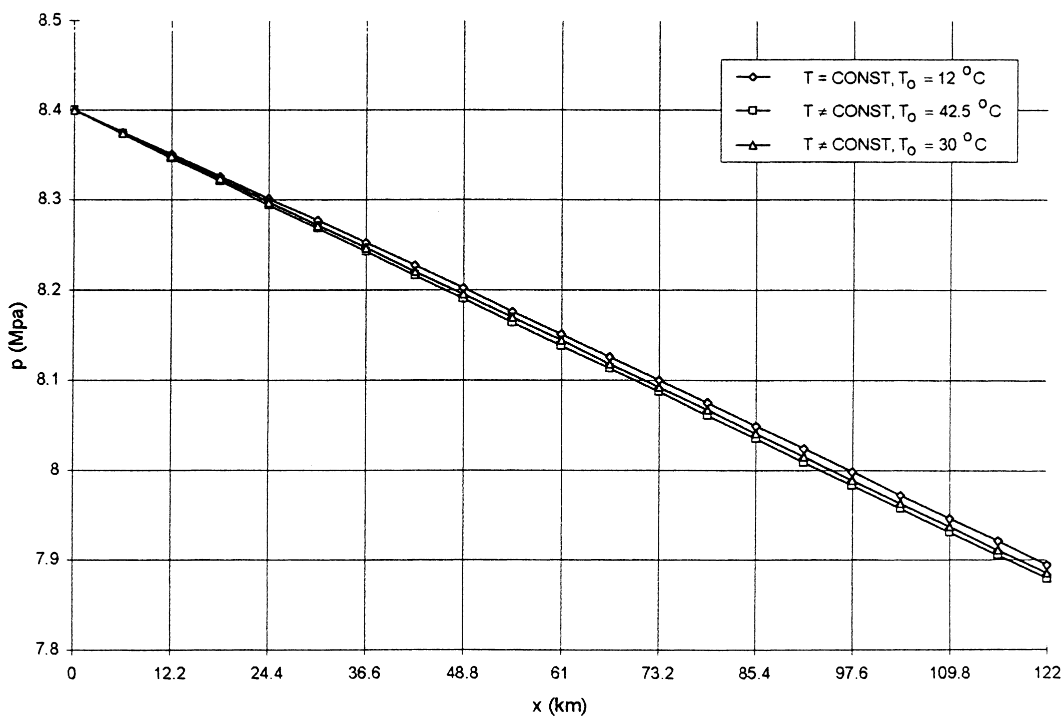

Fig. 3. Change in pressure along the pipeline for (a) T=const, T_0=12°C, (b) $T\neq$const, T_0=42.5°C, (c) $T\neq$const, T_0=30°C.

Table 1
Comparison of capacity of the pipeline for the above-analyzed cases

Discharge temperature (°C)	Capacity (m^3/h)	Capacity difference	
		m^3/h	%
42.5 (without cooling system)	2019949.7	–	–
30.0 (with cooling system)	2032010.2	12060.5	+0.6
12.0 (soil temperature)	2049998.4	30048.7	+1.5

Fig. 4 shows that, for non-isothermal flow, the pressure drop along the pipeline is greater than in the isothermal case. It is a result of the decrease of the density of the gas, allowing the smaller mass of gas to be transported at the specific velocity.

In Fig. 5, the difference in HP used by compressor stations is presented. We can see significant difference in the energy consumption of the drivers of the compressors of each case. It means that the cost of running the transportation system is a function of discharge temperature. The lowest costs correspond to the isothermal flow.

It is clear that, by decreasing the discharge temperature of the gas, the effectiveness of the transportation process can be significantly increased.

4. Transient non-isothermal model

With respect to the conveyance of gas in pipelines, there are two technically relevant extreme cases of pipe flow:

- flow without heat exchange with the ground outside: adiabatic, and more especially, isentropic flow;
- flow with heat exchange with the ground outside, which is regarded as being a heat storage unit of infinite capacity with constant temperature T_0: isothermal flow.

The flow processes of greatest concern here are those in which temperature equalization with the ground outside cannot take place. The temperature profile is a function of pipeline distance. In this case, the transient, non-isothermal flow of gas in a horizontal pipe ($\rho g \sin\alpha=0$, $((\partial/\partial x)(\rho w A g z\, dx))=0$) is described by the system of equations (2)–(5).

Two contradictory constraints are imposed on the above equations. The requirement is that, on the one hand, the description of the phenomenon is accurate, and on the other, there is sufficient simplification to allow the solving of this model by reasonable computation means. As a rule, simplified models are sought which present a reasonable compromise between the accuracy of the description and the costs. The simplified models are obtained by neglecting some terms in the basic (accurate) equation. This results from the quantitative estimation of particular elements of the equation, carried for some given conditions of operation of the pipeline. This means that the model of transient flow used for simulation should be fitted to the given conditions of operation of the pipe. A necessary condition for proper selection of the model is therefore the earlier analysis of these conditions. Estimation of the particular terms of Eq. (5) for given operating conditions and a given geometry of the pipe is given below.

4.1. Energy equation simplifications

Assuming that heat transfer is limited only to conduction through a walled tube and the gas along a pipeline, the

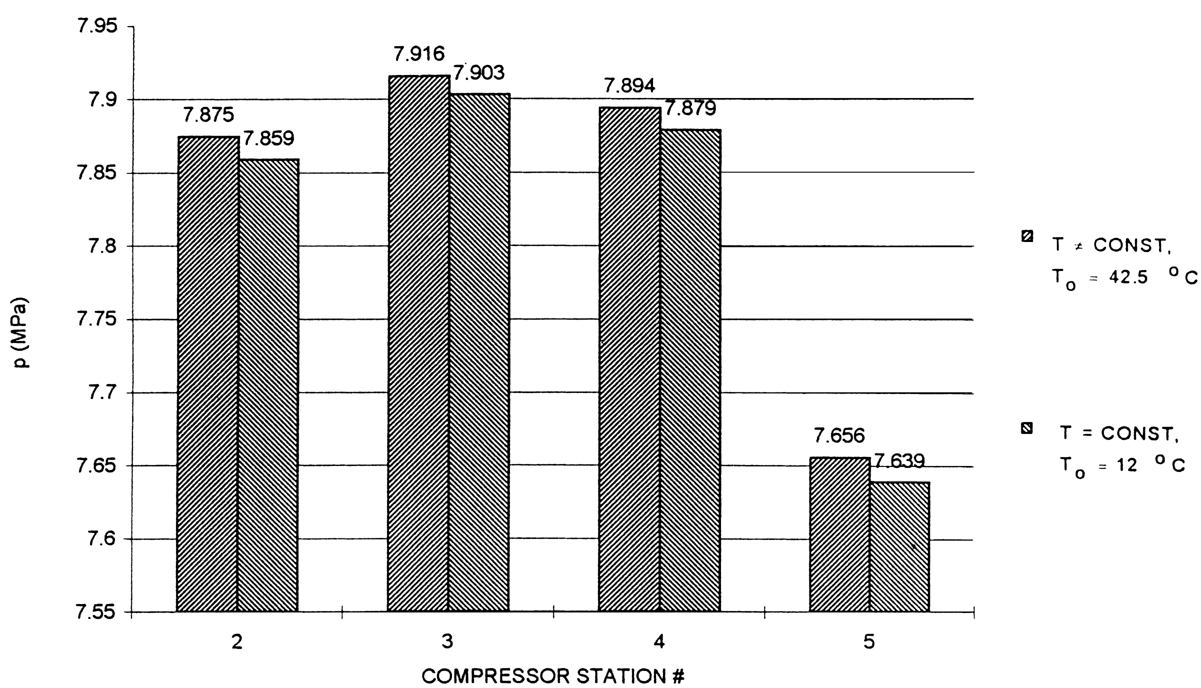

Fig. 4. Comparison of suction pressures for the whole transportation system.

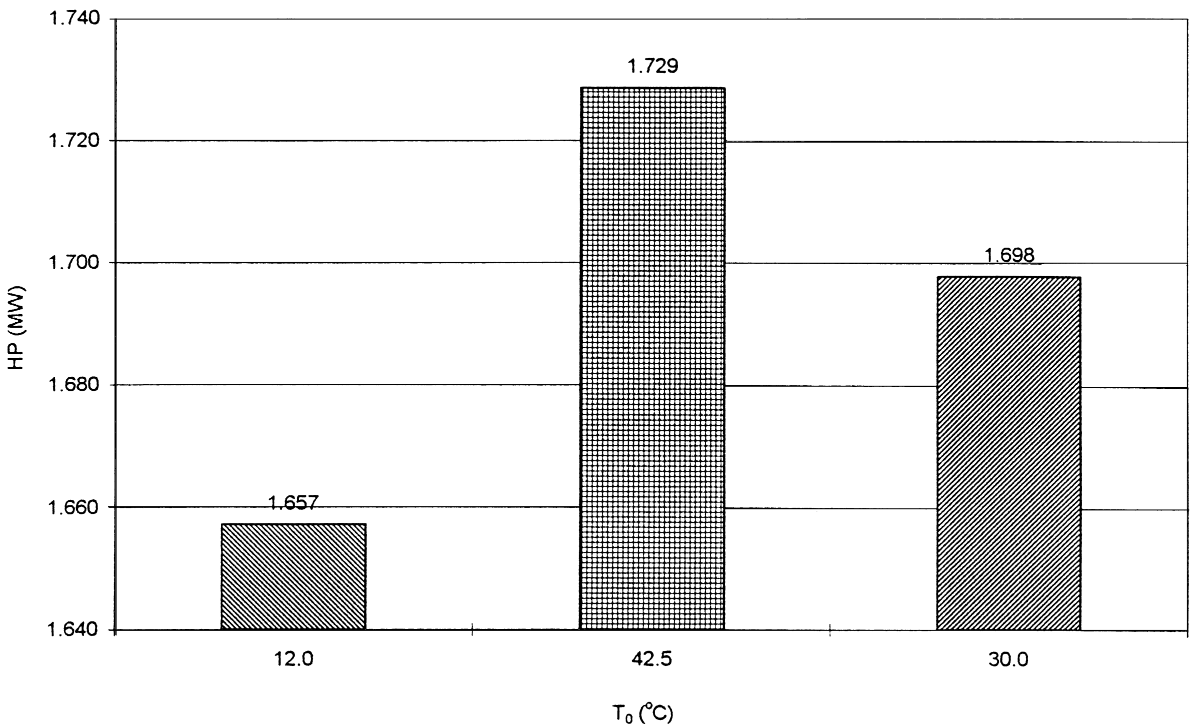

Fig. 5. HP as a function of discharge temperature for the whole transportation system.

following equation can be written:

$$q\rho A\,\mathrm{d}x = A\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)\mathrm{d}x - k_L(T - T_{\mathrm{soil}})\,\mathrm{d}x \qquad (10)$$

where λ is the thermal conductivity coefficient of gas, W/m K; and k_L is the heat transfer coefficient, W/m K.

Combining Eqs. (5) and (10), the final version of the equation can be put into the form

$$\underbrace{\frac{\partial}{\partial x}(\rho w A c_v T\,\mathrm{d}x)}_{\mathrm{I}} + \underbrace{\frac{\partial}{\partial x}\left(\frac{\rho w A p}{\rho}\mathrm{d}x\right)}_{\mathrm{II}} + \underbrace{\frac{\partial}{\partial x}\left(\frac{\rho A w^3}{2}\mathrm{d}x\right)}_{\mathrm{III}} + \underbrace{\frac{\partial}{\partial x}(\rho w A g z\,\mathrm{d}x)}_{\mathrm{IV}} - \underbrace{\frac{\partial}{\partial x}\left(\lambda A\frac{\partial T}{\partial x}\mathrm{d}x\right)}_{\mathrm{V}} + \underbrace{k_L(T - T_{\mathrm{soil}})\,\mathrm{d}x}_{\mathrm{VI}} + \underbrace{\frac{\partial}{\partial t}(\rho A c_v T\,\mathrm{d}x)}_{\mathrm{VII}} + \underbrace{\frac{\partial}{\partial t}\left(\frac{\rho A w^2}{2}\mathrm{d}x\right)}_{\mathrm{VIII}} + \underbrace{\frac{\partial}{\partial t}(\rho A g z\,\mathrm{d}x)}_{\mathrm{IX}} = 0 \qquad (11)$$

By integrating the above equation between x=0 and x=L (where L is the length of the pipe) for the following parameters of the system:

- pipe diameter D=1422 mm, pipe wall thickness 19.2 mm;
- pipeline length L=122 km;
- pressure at x=0 (discharge pressure) p_1=8.4 MPa;
- suction pressure p_2=7.88 MPa;
- temperature at x=0 (discharge temperature) $T|_{x=0}$= 42.5°C;
- temperature at x=L (suction temperature) $T|_{x=L}$=13°C;
- density (under standard conditions) ρ_{n}=0.7156 kg/m^3;
- flow (under standard conditions) Q_{n}=2 019 950 m^3/h;
- soil temperature T_{soil}=12°C;
- heat transfer coefficient k_L=25 W/m K;
- thermal conductivity coefficient of gas λ_{ave}=3.4×10^{-2} W/m K;
- specific gas constant R=518.8 J/kg K;
- specific heat at constant pressure c_p=2.278×10^3 J/kg K;
- specific heat at constant volume c_v=1.759×10^3 J/kg K;

we get the following values for each term of the equation

$$\int_0^L \frac{\partial}{\partial x}(\rho w A c_v T)\,\mathrm{d}x \approx \rho_{\mathrm{n}} Q_{\mathrm{n}} c_v (T|_{x=L} - T|_{x=0})$$
$$= 0.7156 \times \frac{2\,019\,950}{3600} \times 1.759 \times 10^3 \times (286 - 315.5)$$
$$= -2.08 \times 10^7\ \mathrm{W} \quad \to \mathrm{O}(10^7) \qquad (\mathrm{I})$$

$$\int_0^L \frac{\partial}{\partial x}\left(\frac{\rho w A p}{\rho}\right)\mathrm{d}x \approx \rho_{\mathrm{n}} Q_{\mathrm{n}} Z R (T|_{x=L} - T|_{x=0})$$
$$= 0.7156 \times \frac{2\,019\,950}{3600} \times 0.97 \times 518.8 \times (286 - 315.5)$$
$$= -6.0 \times 10^6\ \mathrm{W} \quad \to \mathrm{O}(10^6) \qquad (\mathrm{II})$$

$$\int_0^L \frac{\partial}{\partial x}\left(\frac{\rho A w^3}{2}\right)\mathrm{d}x \approx \frac{1}{2}\rho_\mathrm{n} Q_\mathrm{n}(w^2|_{x=L} - w^2|_{x=0})$$
$$= \frac{1}{2}\times 0.7156 \times \frac{2\,019\,950}{3600}\times(4.96^2 - 5.13^2)$$
$$= -3.4\times 10^2\,\mathrm{W} \quad \to \mathrm{O}(10^2) \qquad \text{(III)}$$

$$\int_0^L \frac{\partial}{\partial x}(\rho w A g z)\,\mathrm{d}x \approx \rho_\mathrm{n} Q_\mathrm{n} g(z|_{x=L} - z|_{x=0}) \qquad \text{(IV)}$$

Assuming that the difference of levels is, e.g. 100 m, we get

$$\rho_\mathrm{n} Q_\mathrm{n} g(z|_{x=L} - z|_{x=0}) = 0.7156\times\frac{2\,019\,950}{3600}\times 9.81\times 100$$
$$= 3.9\times 10^5\,\mathrm{W} \quad \to \mathrm{O}(10^5)$$

$$-\int_0^L \left[\frac{\partial}{\partial x}\left(\lambda A\frac{\partial T}{\partial x}\right)\right]\mathrm{d}x \approx -\lambda_\mathrm{ave} A\frac{\Delta T|_{x=L} - \Delta T|_{x=0}}{\Delta x} \qquad \text{(V)}$$

from the steady-state analysis, we have

$$\left.\frac{\Delta T}{\Delta x}\right|_{x=0} = \frac{4.8}{6100}\,\mathrm{K/m}$$
$$\left.\frac{\Delta T}{\Delta x}\right|_{x=L} = \frac{0.2}{6100}\,\mathrm{K/m}$$

$$-\lambda_\mathrm{ave} A\frac{\Delta T|_{x=L} - \Delta T|_{x=0}}{\Delta x}$$
$$= -3.4\times 10^{-2}\times\frac{\pi\times 1.3836^2}{4}\times\frac{4.8-0.2}{6100}$$
$$= 3.9\times 10^{-5}\,\mathrm{W} \quad \to \mathrm{O}(10^{-5})$$

$$\int_0^L k_L(T - T_\mathrm{soil})\,\mathrm{d}x \approx k_L L\,\Delta T_\mathrm{s} \qquad \text{(VI)}$$

Substituting the mean temperature difference between the gas and the soil, ΔT_s=15.75 K, we have

$$k_L L\,\Delta T_\mathrm{s} = 25\times 1.22\times 10^5\times 15.75$$
$$= 4.8\times 10^7\,\mathrm{W} \quad \to \mathrm{O}(10^7)$$

$$\int_0^L \frac{\partial}{\partial t}(\rho A c_v T)\,\mathrm{d}x \approx \rho A c_v L\frac{\Delta T}{\Delta t} \qquad \text{(VII)}$$

Let the increment of the temperature be 5 K and let it be achieved within Δt=1 h. Substituting the mean value of gas density along the pipeline, $\bar{\rho} = 64.27\,\mathrm{kg/m^3}$, we get

$$\bar{\rho} A c_v L\frac{\Delta T}{\Delta t} = 64.27\times\frac{\pi\times 1.3836^2}{4}\times 1.759\times 10^3$$
$$\times 1.22\times 10^5\times\frac{5}{3600} = 2.88\times 10^7\,\mathrm{W}$$
$$\to \mathrm{O}(10^7)$$

$$\int_0^L \frac{\partial}{\partial t}\left(\frac{\rho A w^2}{2}\right)\mathrm{d}x \approx \frac{L\bar{w}}{2}\frac{\partial(\rho A w)}{\partial t} \approx \frac{L\bar{w}\rho_\mathrm{n}}{2}\frac{\Delta Q_\mathrm{n}}{\Delta t} \qquad \text{(VIII)}$$

Let the increment of the load of the pipeline at x=L be $0.5Q_\mathrm{n}$ and let it be achieved within Δt=1 h. We then get

$$\frac{L\bar{w}\rho_\mathrm{n}}{2}\frac{\Delta Q_\mathrm{n}}{\Delta t} = \frac{1.22\times 10^5\times 5.04\times 0.7156}{2}\times\frac{1\,009\,975}{3600}$$
$$= 6.18\times 10^7\,\mathrm{W} \quad \to \mathrm{O}(10^7)$$

$$\int_0^L \frac{\partial}{\partial t}(\rho g z A)\,\mathrm{d}x \approx \rho g A L\frac{\partial z}{\partial t} = 0 \quad \text{(for each gas pipeline)} \qquad \text{(IX)}$$

Based on above-given analysis, we can write the simplified form of the energy equation:

$$k_L(T_\mathrm{soil} - T)\,\mathrm{d}x = \frac{\partial}{\partial t}\left[(\rho A\,\mathrm{d}x)\left(u + \frac{w^2}{2}\right)\right] + \frac{\partial}{\partial x}\left[(\rho w A\,\mathrm{d}x)\left(u + \frac{p}{\rho}\right)\right]$$

The character of the results cannot be generalized. This can only be the starting point, which allows the forwarding of the hypothesis that, in the case when the selected parameters do not change rapidly, transient non-isothermal flow through the horizontal pipe can be represented be the set of equations in the form

$$\begin{cases} -\dfrac{\partial(\rho w)}{\partial x} = \dfrac{\partial \rho}{\partial t} \\ -\dfrac{\partial p}{\partial x} - \dfrac{2f\rho w^2}{D} = \dfrac{\partial(\rho w)}{\partial t} + \dfrac{\partial(\rho w^2)}{\partial x} \\ k_L(T_\mathrm{soil} - T)\,\mathrm{d}x = \dfrac{\partial}{\partial t}\left[(\rho A\,\mathrm{d}x)\left(u + \dfrac{w^2}{2}\right)\right] \\ \qquad + \dfrac{\partial}{\partial x}\left[(\rho w A\,\mathrm{d}x)\left(u + \dfrac{p}{\rho}\right)\right] \\ \dfrac{p}{\rho} = ZRT \end{cases}$$

4.2. Transient simulation

The investigations were carried out using the method of lines [9] for the following values of parameters (Fig. 6):

- pipe diameter D=1422 mm, pipe wall thickness 19.2 mm;
- pipeline length L=122 km;
- temperature at x=0 (discharge temperature) $T|_{x=0}$= 42.5°C;
- density (under standard conditions) ρ_n=0.7156 kg/m^3;
- soil temperature T_soil=12°C;
- heat transfer coefficient k_L=25 W/m K;
- thermal conductivity coefficient of gas λ_ave=3.4×10^{-2} W/m K;
- specific gas constant R=518.8 J/kg K;
- specific heat at constant pressure c_p=2.278×10^3 J/kg K;
- specific heat at constant volume c_v=1.759×10^3 J/kg K;

$$\left.\begin{aligned} p(0,t) &= \text{const} = 8.4\,\mathrm{MPa} \\ Q_\mathrm{n}(L,t) &= f(t) \end{aligned}\right\}\ \text{boundary conditions}$$

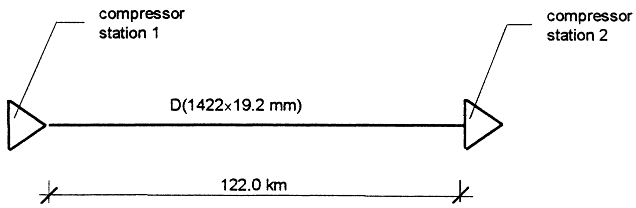

Fig. 6. Structure of the gas transportation system.

The function $Q_n(L,t)=f(t)$ is shown in Fig. 7. For a given range of variation of the load, flow behavior in the pipeline is approximated accurately by the rough-pipe flow law [1]. This law is represented by the horizontal lines in the Moody diagram of friction factor, with each line corresponding to a specific value of the relative roughness of the pipe.

Results of calculation are presented in Figs. 8–12.

Maximum pressure difference between non-isothermal flow (without cooling system) and isothermal flow is given in the following equation:

$$\delta_{\max} = \frac{p_{\text{isotherm}}|_{x=J} - p_{\text{n-isotherm}}|_{x=J}}{p_{\text{isotherm}}|_{x=J}} \times 100\% = \frac{7.145 - 7.067}{7.145} \times 100 = 1.09\%$$

Maximum pressure difference between non-isothermal flow (without cooling system) and non-isothermal flow (with cooling system) is given in the following equation:

$$\delta_{\max} = \frac{p_{\text{isotherm}}|_{x=J} - p_{\text{n-isotherm}}|_{x=J}}{p_{\text{isotherm}}|_{x=J}} \times 100\% = \frac{7.145 - 7.117}{7.145} \times 100 = 0.39\%$$

Profile of temperature at $x=L$ is caused by load variations and by distributed velocity of the gas along the pipeline.

Maximum compressor ratio difference between non-isothermal flow (without cooling system) and isothermal flow is given in the following equation:

$$|\Delta\varepsilon_{\max}| = \left|\frac{\varepsilon_{\text{isotherm}} - \varepsilon_{\text{n-isotherm}}}{\varepsilon_{\text{isotherm}}}\right| \times 100\% = \left|\frac{1.18 - 1.26}{1.18}\right| \times 100 = 6.78\%$$

Maximum compressor ratio difference between non-isothermal flow (with cooling system): and isothermal flow is given in the following equation:

$$|\Delta\varepsilon_{\max}| = \left|\frac{\varepsilon_{\text{isotherm}} - \varepsilon_{\text{n-isotherm}}}{\varepsilon_{\text{isotherm}}}\right| \times 100\% = \left|\frac{1.074 - 1.078}{1.074}\right| \times 100 = 0.37\%$$

Maximum HP difference necessary to keep constant discharge pressure at compressor station no. 2 (Fig. 6) for non-isothermal flow (without cooling system) and isother-

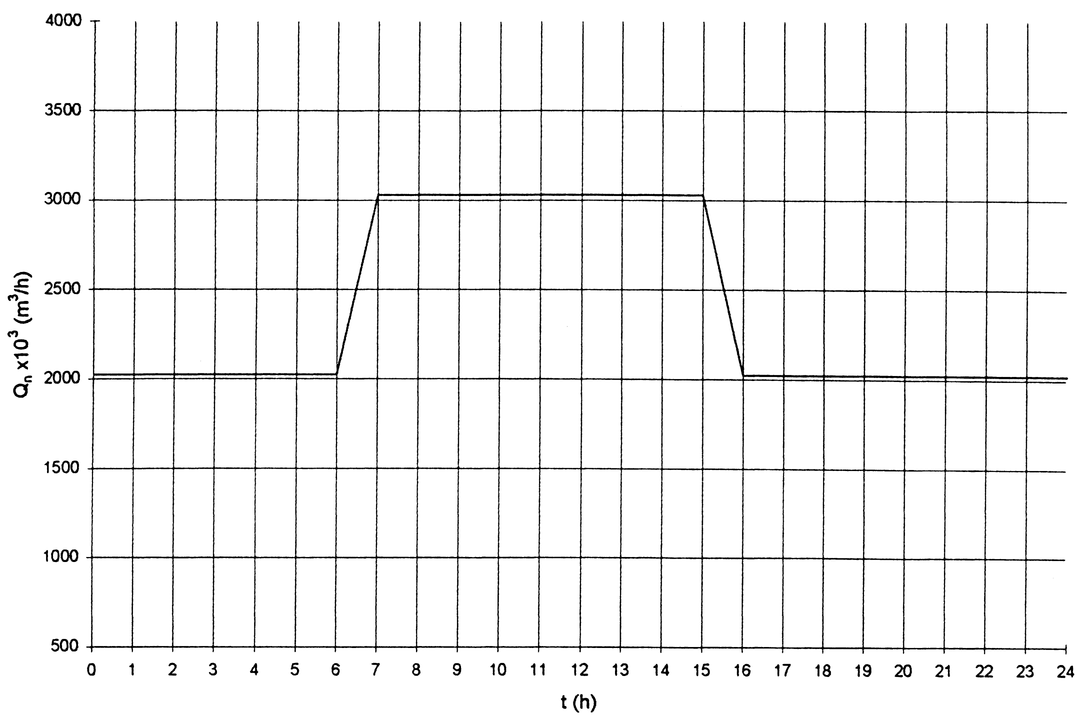

Fig. 7. Change in flow with time ($x=L$) — boundary condition.

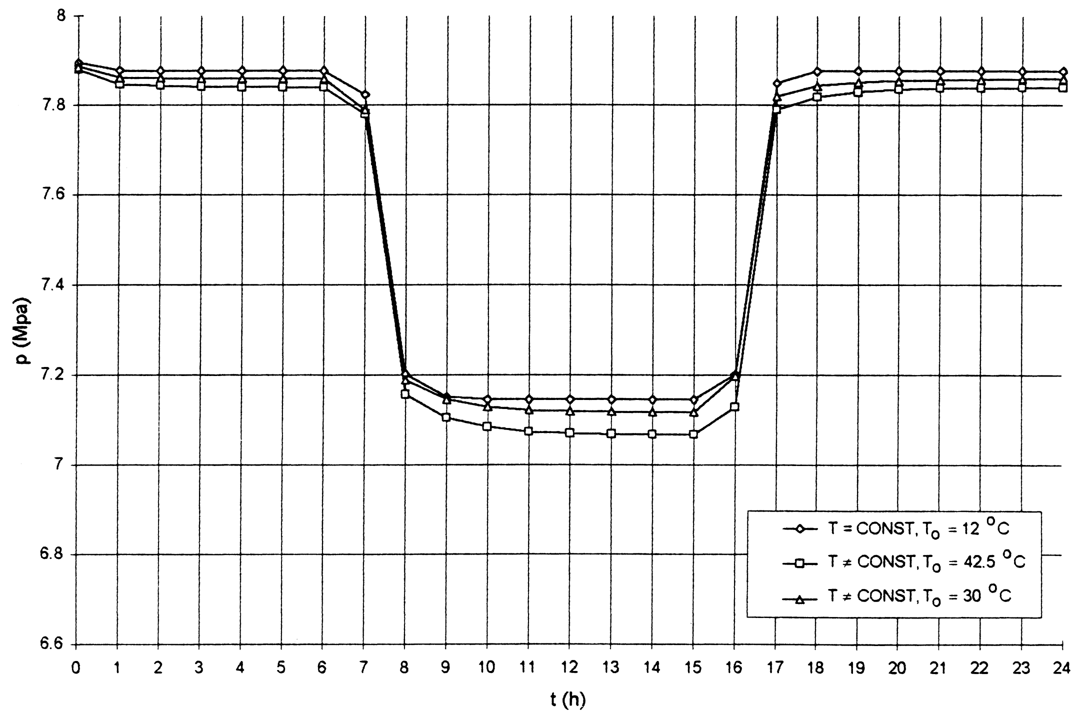

Fig. 8. Change in pressure at $x=L$.

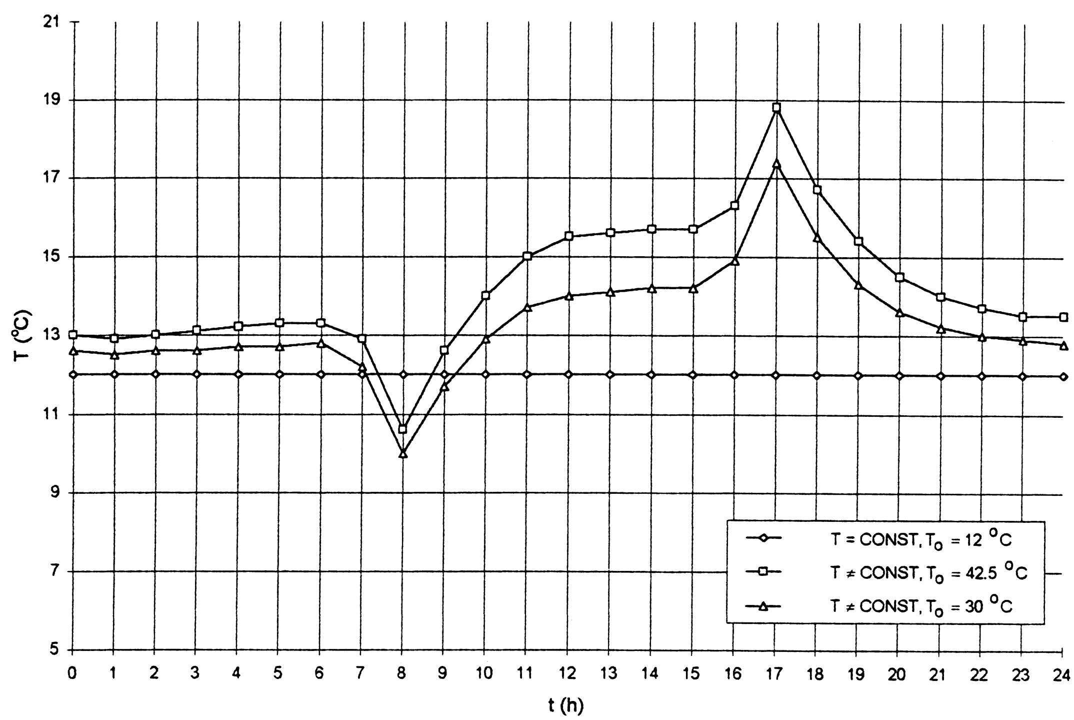

Fig. 9. Change in temperature at $x=L$.

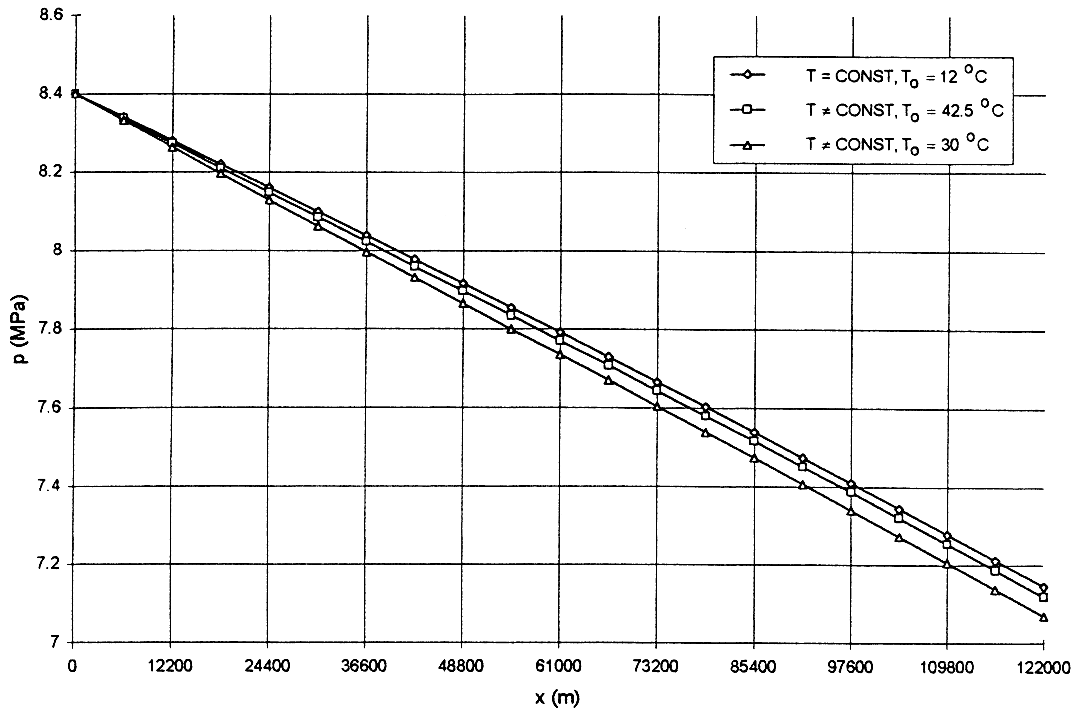

Fig. 10. Change in pressure along the pipeline for t=12 h.

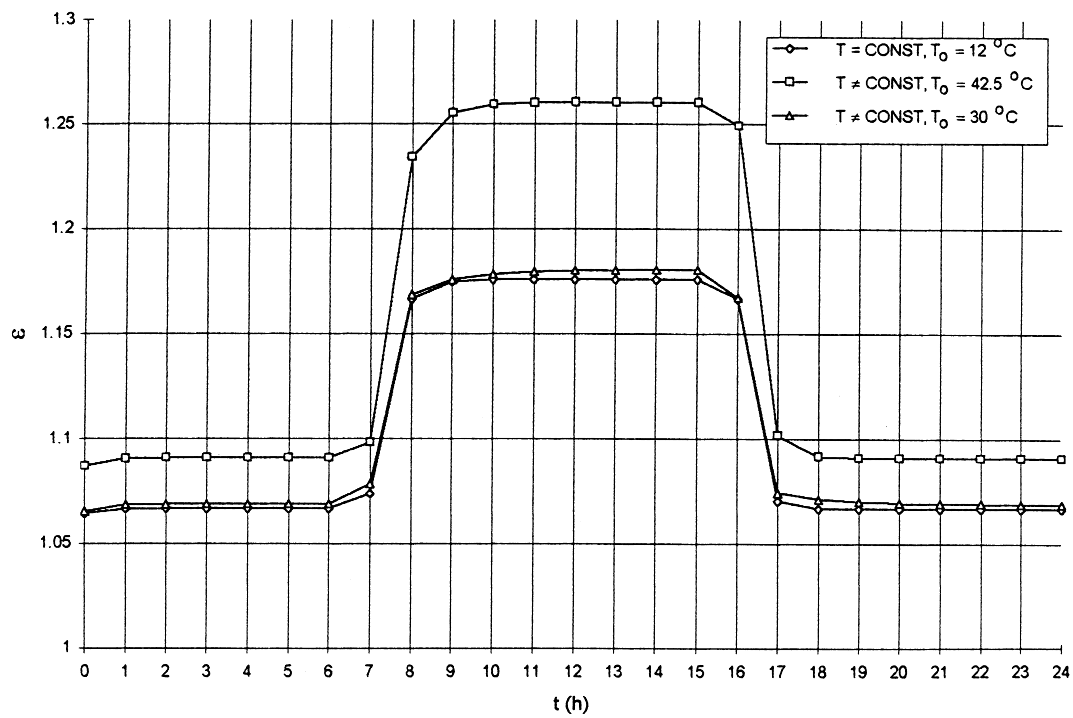

Fig. 11. Change in compressor ratio at compressor station no. 2.

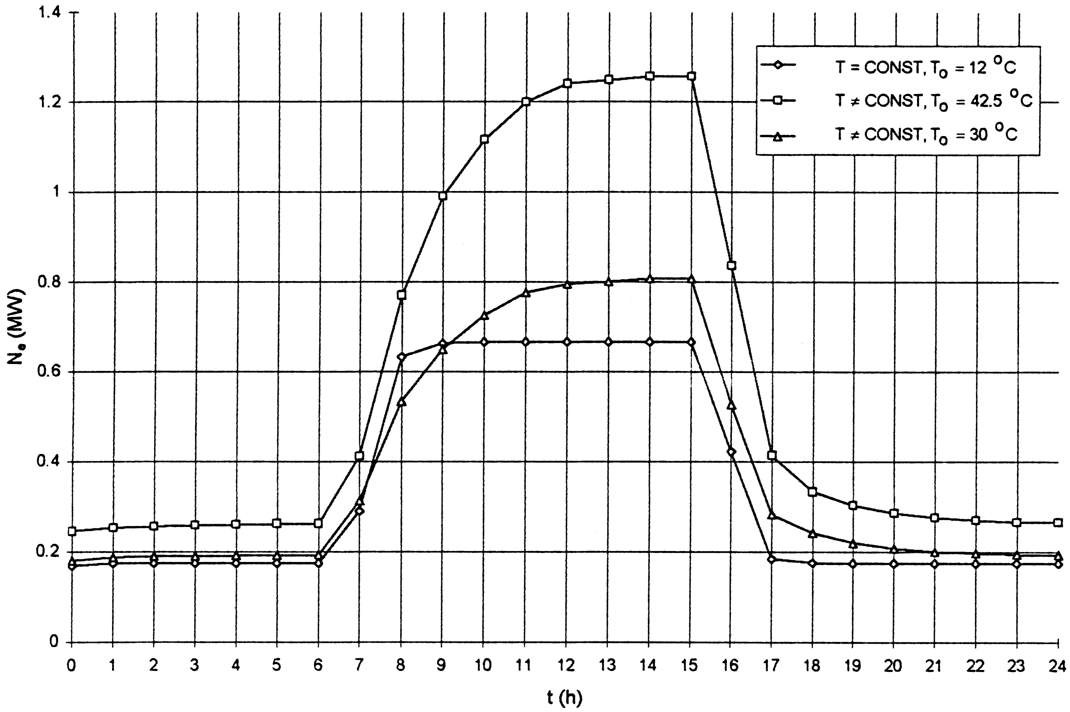

Fig. 12. Changes in HP at compressor station no. 2 for the isentropic compression.

mal flow is given in the following equation:

$$|\Delta N_{\max}| = \left|\frac{N_{\text{isotherm}} - N_{\text{n-isotherm}}}{N_{\text{isotherm}}}\right| \times 100\% = \left|\frac{0.666 - 1.255}{0.666}\right| \times 100 = 88.43\%$$

Maximum HP difference necessary to keep constant the discharge pressure at compressor station no. 2 (Fig. 6) for non-isothermal flow (with cooling system) and isothermal flow is given in the following equation:

$$|\Delta N_{\max}| = \left|\frac{N_{\text{isotherm}} - N_{\text{n-isotherm}}}{N_{\text{isotherm}}}\right| \times 100\% = \left|\frac{0.666 - 0.807}{0.666}\right| \times 100 = 21.17\%$$

5. Conclusions

It is clear that cooling of the gas improves the efficiency of the overall compression process. There exists a significant difference in the pressure profile along the pipeline between isothermal and non-isothermal process. This difference increases when the quantity of gas increases. This shows that, in the case when gas temperature does not stabilize, the use of an isothermal model leads to significant errors. The problem of choosing the correct model is a function of network structure and network complexity.

References

[1] The American Gas Association, Distribution-System Design, AGA, Arlington, 1990.
[2] H. Daneshyar, One-dimensional Compressible Flow, Pergamon Press, New York, 1976.
[3] M.H. Goldwater, A.E. Fincham, in: H. Nicholson (Ed.), Modelling of Dynamical Systems, Peter Peregrinus, Stevenage, 1981.
[4] J.P. Holman, Thermodynamics, McGraw-Hill, New York, 1980.
[5] ISO 12213-3, Natural gas — calculation of compression factor. Part 3: Calculation using physical properties. International Organization for Standarization, Technical Committee ISO/TC 193, Natural gas, Sub committee SC1, Analysis of natural gas. Geneve, 1997.
[6] J. Kralik, et al., Dynamic Modelling of Large Scale Networks With Application to Gas Distribution, Elsevier, New York, 1988.
[7] A.J. Osiadacz, Simulation and Analysis of Gas Networks, Gulf Publishing Company, Houston, 1989.
[8] A.J. Osiadacz, Different transient models — limitations, advantages and disadvantages, in: Proceedings of the PSIG — The 28th Annual Meeting, San Francisco, 1996.
[9] W.E. Schiesser, The Numerical Methods of Lines, Academic Press, London, 1991.
[10] A.H. Shapiro, The Dynamics and Thermodynamics of Compressible Fluid Flow, The Ronald Press Company, New York, 1954.
[11] E.B. Wylie, V.L. Streeter, Fluid Transients, McGraw-Hill, New York, 1978.